REALIZATION OF NONLINEAR OSCILLATORS WITH A TRAPPED ION *

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Abstract

We consider a trapped ion with a quantized center–of–mass motion in 2D trap potential. With external laser fields the effective (non)linear coupling of two orthogonal vibrational modes can be established via stimulated Raman transition. Nonclassical vibrational states such as squeezed states or two–mode entangled states (Schrödinger cat–like states) can be generated. When the vibrational modes are entangled with internal energy levels the Greenberger–Horne–Zeilinger (GHZ) states can be prepared.

I. MOTIVATION

The Jaynes–Cummings model (JCM) [1] which describes an interaction of one two–level atom with a quantized cavity field belongs to the fundamentals of quantum optics. The successful experimental realization of the JCM is

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associated with Rydberg atoms in a high-Q microwave cavity. The quantum effects such as the collapse–revival behavior and preparation of Schrödinger cat–like states have been demonstrated [2].

Recent experimental developments in laser cooling and ion trapping [3] have enabled to realize a formal analogue of the JCM in which the cavity field mode is replaced by a quantized vibrational mode of the center–of–mass motion of a trapped ion [4–6]. There are two virtues in experiments with trapped ions. Firstly, dissipative effects which are inevitable from cavity damping in the microwave or optical domain can be significantly suppressed for the ion motion. Secondly, the motion of the trapped ion can be well controlled by proper sequences of laser pulses tuned either to the atomic electronic transition or to resolved vibrational sidebands. These aspects make trapped ions candidates for quantum computing [7]. It is worth noticing that the most prominent nonclassical vibrational states of a trapped ion (coherent, squeezed, Fock and Schrödinger cat–like states) have been successfully prepared in laboratory [5].

The formal analogy of the vibrational mode of a trapped ion with the cavity field mode can be extended to multi–mode systems. In particular, schemes for preparing correlated two–mode Schrödinger cat states, the Bell and SU(2) states of vibrational motion in a two–dimensional trap have been proposed [8].

In this paper we consider a trapped ion with a quantized center–of–mass motion in 2D trap potential. Irradiating the ion with two external laser fields, which are tuned to well–resolved vibrational sidebands to stimulate Raman transition between internal energy levels, the effective linear or nonlinear coupling of two orthogonal vibrational modes can be established. It is shown how to realize with a trapped ion also analogues of other fundamental elements of quantum optics - nonlinear optical processes (multiwave mixings) such as the

degenerate two–photon down conversion.

II. THE MODELS

Consider a quantized center–of–mass motion of an ion which is confined in a 2D harmonic potential characterized by the trap frequencies ν_x and ν_y in two orthogonal directions x and y. The ion is irradiated by two external laser fields with frequencies ω_x, ω_y along the x and y axes. The laser fields stimulate transitions between three internal energy levels a, b, c in Λ configuration (with the upper c level). The interaction Hamiltonian for the system under consideration can be written in the form:

$$\begin{aligned} \hat{H}_{int} = & \frac{1}{2}\hbar\Omega_x \left[\mathrm{e}^{-i\omega_x t}\hat{D}_x(i\epsilon_x)|c\rangle\langle a| + \mathrm{e}^{i\omega_x t}\hat{D}_x(-i\epsilon_x)|a\rangle\langle c| \right] \\ & + \frac{1}{2}\hbar\Omega_y \left[\mathrm{e}^{-i\omega_y t}\hat{D}_y(i\epsilon_y)|c\rangle\langle b| + \mathrm{e}^{i\omega_y t}\hat{D}_y(-i\epsilon_y)|b\rangle\langle c| \right] . \end{aligned} \tag{1}$$

The absorption (emission) of energy from (to) external laser field in q direction ($q = x, y$) is accompanied by momentum exchange between the field and the ion which is described in (1) by the displacement operator $\hat{D}_q(i\epsilon_q) = \exp[i\epsilon_q(\hat{a}_q^\dagger + \hat{a}_q)]$; $\hat{a}_q^\dagger$, $\hat{a}_q$ are the creation and annihilation operators of vibrational quanta in a given direction. The parameter ϵ_q is defined as $\epsilon_q^2 = E_q^{(r)}/(\hbar\nu_q)$ where $E_q^{(r)}$ is the classical recoil energy of the ion; Ω_q is a Rabi frequency of the laser–driven transition and $|i\rangle\langle j|$ is an atomic transition operator.

In the Lamb-Dicke limit $\epsilon_x \approx \epsilon_y \ll 1$ only resonant processes can be taken into account. The upper c level can be adiabatically eliminated under resonance conditions on the frequencies of the laser fields:

$$E_a/\hbar + \omega_x + m\nu_x = E_b/\hbar + \omega_y + n\nu_y \tag{2}$$

(i.e., m and n trap quanta are involved in the stimulated Raman transition between internal energy levels a and b) and off–resonance conditions for the

transitions from levels a and b to the upper level c (with detuning $\Delta \gg m\nu_x, n\nu_y$):

$$E_c - E_a = \hbar\omega_x + m\hbar\nu_x + \hbar\Delta, \qquad E_c - E_b = \hbar\omega_y + n\hbar\nu_y + \hbar\Delta. \tag{3}$$

The effective interaction Hamiltonian in the rotating–wave-approximation and the interaction picture reads:

$$\begin{aligned} \hat{H}_{eff} = & -\left[\frac{\hbar(-1)^n(i\epsilon)^{m+n}\Omega_x\Omega_y}{4m!n!\Delta}\hat{a}_x^m\hat{a}_y^{\dagger n}|b\rangle\langle a| + h.c.\right] \\ & -\left[\frac{\hbar\epsilon^{2m}\Omega_x^2}{4m!^2\Delta}\hat{a}_x^m\hat{a}_x^{\dagger m}|a\rangle\langle a| + \frac{\hbar\epsilon^{2n}\Omega_y^2}{4n!^2\Delta}\hat{a}_y^n\hat{a}_y^{\dagger n}|b\rangle\langle b|\right]. \end{aligned} \tag{4}$$

The terms in the second line represent generalized Stark shifts of the levels a and b.

For example, for $m = n = 1$ one gets $\hat{H}_{eff} = \hbar\lambda(\hat{a}_x\hat{a}_y^\dagger|b\rangle\langle a| + \hat{a}_x^\dagger\hat{a}_y|a\rangle\langle b|)$. This exactly solvable interaction Hamiltonian is known in the cavity QED within a context of two-photon transitions of a two-level atom interacting with a bichromatic field in a cavity as well for an ion in 1D trap with a resonator [9]. The mutual entanglement (correlation) of two vibrational modes with ionic internal degrees of freedom is established. When the vibrational modes x, y are initially prepared in coherent states $|\beta\rangle_x, |\gamma\rangle_y$, the dynamics of the vibrational modes x, y in phase space is characterized by revival times $t_R^{(x)} \approx \frac{2\pi\beta}{\lambda\gamma}$, $t_R^{(y)} \approx \frac{2\pi\gamma}{\lambda\beta}$ (i.e., the initial quasidistributions are partially restored). If $\beta \approx \gamma$ the revival times are independent of the coherent amplitudes and a collapse–revival structure of the atomic inversion can be observed. Similarly to the JCM, the quasidistributions bifurcate into two components. Nevertheless, the vibrational modes are entangled with the internal energy levels. To disentangle them, conditional measurements on the internal energy levels can be performed [10]. For example, the internal state of the ion can be determined by driving transition from the level b to an auxiliary level d and observing the fluorescence signal. No signal (no interaction with probing

field) means the undisturbed ion in the level a. One can thus prepare *pure* two–mode correlated states of the vibrational system. At half of the revival time $t_R^{(x)} \approx t_R^{(y)}$ one finds a structure close to a pure two–mode Schrödinger cat–like state which consists of four components. Each particular vibrational mode is in a two–component mixture (the components are mutually rotated by π in the phase space).

It is worth noticing that GHZ states of importance in tests of quantum mechanics can be generated in a rather simple way when the vibrational modes are still entangled with internal energy levels. Preparing the initial state with one trap quantum in x mode, i.e., $|\psi(0)\rangle = |1\rangle_x|0\rangle_y|a\rangle$, one gets at quarter of the Rabi cycle $\lambda t = \pi/4$ the GHZ state $|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_x|0\rangle_y|a\rangle - i|0\rangle_x|1\rangle_y|b\rangle)$. If instead of (2) the resonance condition $E_a/\hbar + \omega_x + \nu_x = E_b/\hbar + \omega_y - \nu_y$ is assumed, one can start with vibrational modes in the vacuum and the ion in the level b to produce a GHZ state.

In the degenerate case, when the level b is identical with the level a all the atomic operators can be eliminated (the c level is far of resonance) and the effective interaction Hamiltonian takes the form

$$\begin{aligned} \hat{H}_{eff}^{(xy)} &= -\left[\frac{\hbar(-1)^n(i\epsilon)^{m+n}\Omega_x\Omega_y}{4m!n!\Delta}\hat{a}_x^m\hat{a}_y^{\dagger n} + h.c.\right] \\ &\quad -\left[\frac{\hbar\epsilon^{2m}\Omega_x^2}{4m!^2\Delta}\hat{a}_x^m\hat{a}_x^{\dagger m} + \frac{\hbar\epsilon^{2n}\Omega_y^2}{4n!^2\Delta}\hat{a}_y^n\hat{a}_y^{\dagger n}\right]. \end{aligned} \tag{5}$$

For $m = n = 1$ the linear coupler is obtained. Nontrivial case - the nonlinear coupling of the bosonic modes - is realized when one of the lasers is tuned to the second vibrational sideband. For example $m = 1$, $n = 2$ leads to two–phonon down conversion governed by $\hat{H}_{eff}^{(xy)} = \hbar\lambda_2(\hat{a}_x\hat{a}_y^{\dagger 2} + \hat{a}_x^\dagger\hat{a}_y^2)$. One can thus simulate the degenerate three–wave mixing [11] even on the long time–scale. It is well–known that starting with coherently excited x and empty y mode the squeezed vacuum in the latter is produced. Besides of that at the time of the maximal depletion of the x mode the oscillations in

photon number distributions of both modes can be observed. In the y mode a two–component mixture is established with a remarkable interference pattern between the components (peaks) of the Wigner function [12].

For experimentalists also the case $m = 1$, $n = 3$ can be of interest. In the three–phonon down conversion a three–fold structure is formed in the phase space of the initially empty y mode. In its Wigner function strong interference structures between three "arms" (rotated by $2\pi/3$ in phase space) are rapidly developed. The initially coherent x mode undergoes radial splitting into two components. The resulting *mixture* exhibits the features typical for *pure* Schrödinger cat states (significant interference pattern in the Wigner function and oscillations of photon number distribution) [12].

III. CONCLUSION

To conclude, the formal analogy of the vibrational mode of a trapped ion with the cavity field mode in the JCM leads to the possibility of realizing some cavity QED ideas without using an optical cavity. We showed that the trapped ion can serve in the Lamb–Dicke limit as a simulator of nonlinear optical processes, namely multiwave mixings, without crucial limitations on the interaction length. The damping of bosonic vibrational modes can be significantly suppressed. The dominant decoherence effect is the spontaneous emission of the ion which is not taken into account here.

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